Chapter 5
Digital PID control algorithm

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Outline

- Abstract
- Quasi-continuous PID control algorithm
- Improvement of standard PID algorithm
- Choosing parameter of PID regulator
- Brief summary

Outline

- The PID（ Proportional - Integral - Differential ） regulator control depending on the proportional, integral and differential of the deviation
- PID regulation is the most mature and the most widely used technology of continuous system. The substance of its regulation is based on the deviation of the input value, a function of the proportional, integral and differential operator， The result of the calculation for the output to control.
- In practical applications, depending on the circumstances, the structure of the PID control can be flexibly changed.

Outline （2）

- The advantage of PID

  ★ mature technology

  ★ Easily familiar with and master

  ★ Do not need to create a mathematical model

  ★ Good control performance

Outline （3）

- To realize PID control
- Analog: electronic circuit regulator, the measured signal compared with a given value, then the difference after PID circuit operation is sent to the actuator, change the amount of input to achieve the purpose of regulation.

Digital: using a computer, the result of the calculation is converted to the analog output to control the actuators.

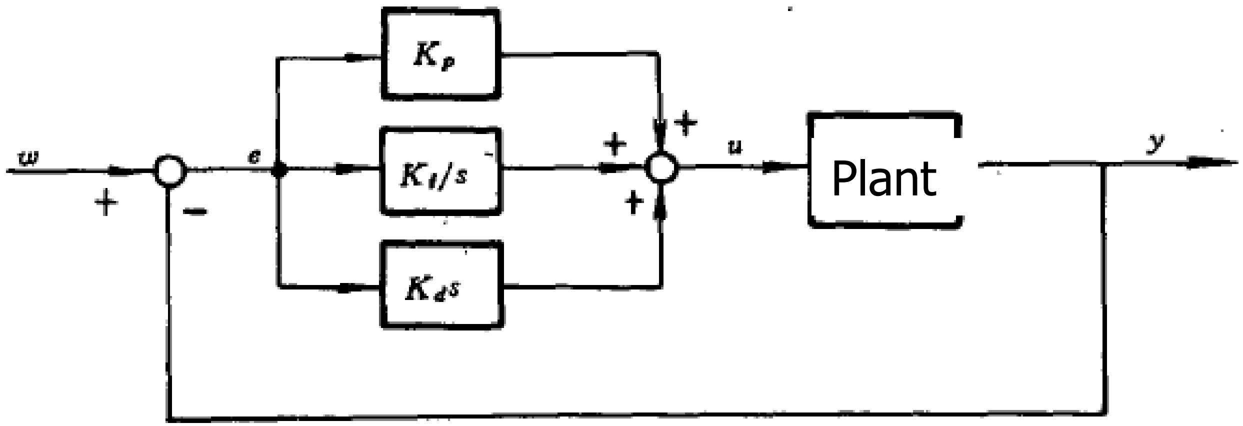
- The regulator design issues. ---------.
- Terminal controller design problem ---.

$$0 \xrightarrow{Disturbance} x_0 \xrightarrow{Control--u} 0$$

$$x_0 \xrightarrow{Control--u} x_f$$

Quasi-continuous PID control algorithm

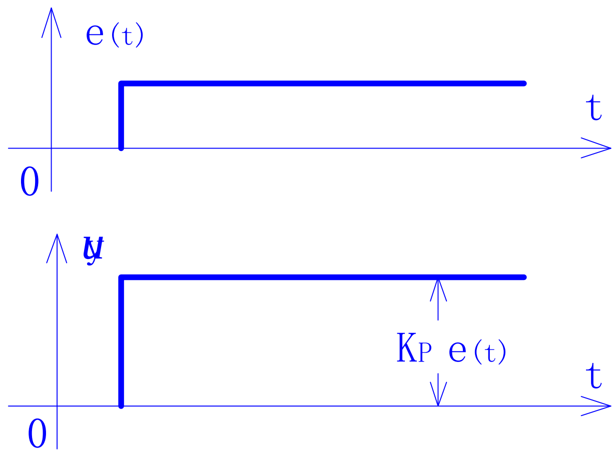
- Analog PID regulator



Quasi-continuous PID control algorithm（2）

- Proportional regulator

$$u = K_P e + u_0$$



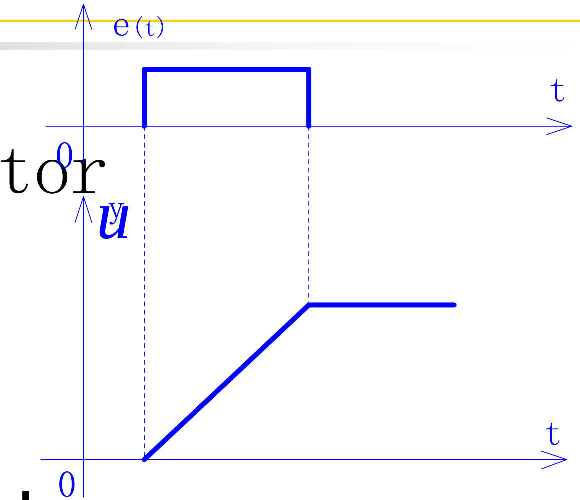
- u — output
- K_P — Scale factor
- e — input deviation
- u_0 — basis of control input

Proportional action: rapid response error, but does not eliminate the steady state error, easily lead to instability if it is too large

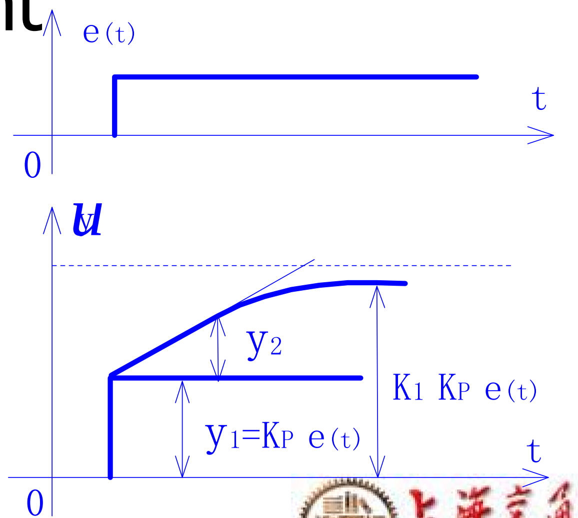
Quasi-continuous PID control algorithm（3）

- Proportional integral regulator

$$u = K_P\left(e + \frac{1}{T_I}\int_0^t e\mathrm{d}t\right) + u_0$$



T_I — Integral time constant



Integral action: eliminate static error,
but may cause overshoot easily,
or even oscillation

Quasi-continuous PID control algorithm（4）

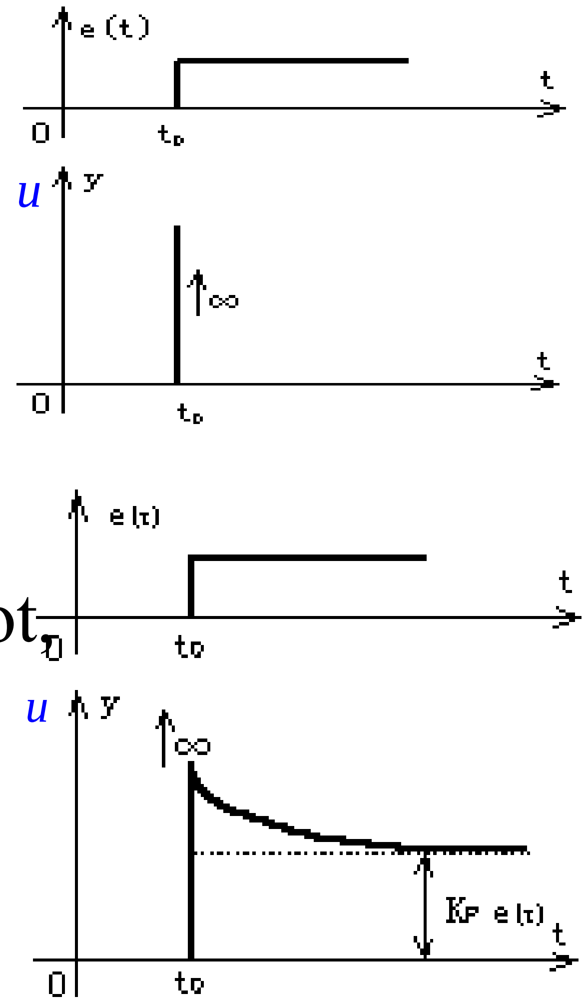
- proportional and differential regulator

$$u = K_P\left(e + T_D\frac{\mathrm{d}e}{\mathrm{d}t}\right) + u_0$$

T_D — Derivative time constant

Derivative action: reduce the overshoot, to overcome the oscillation

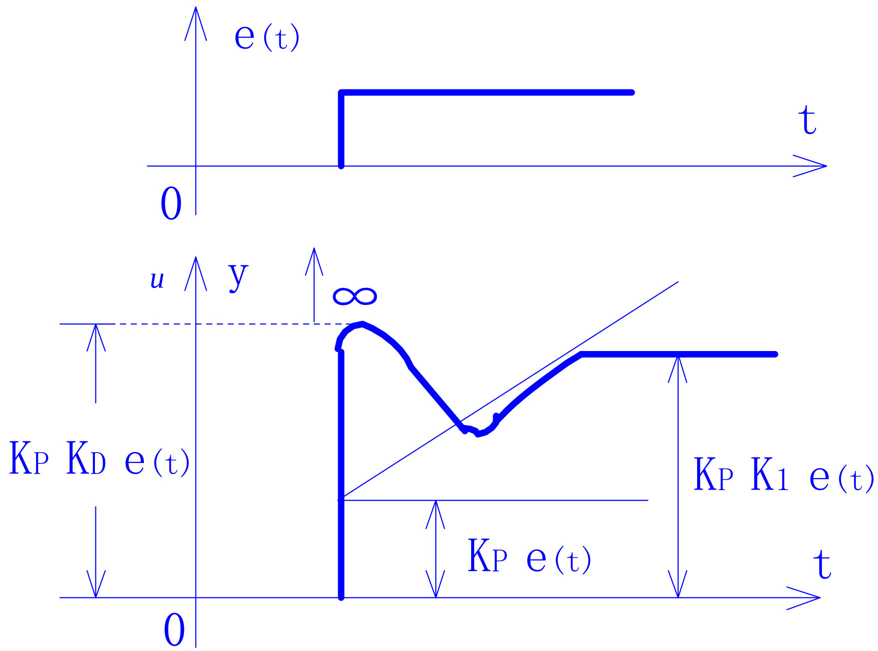
Improve stability, to improve the system dynamic characteristics



Quasi-continuous PID control algorithm（5）

- Proportional integral differential regulator

$$u = K_P\left(e + \frac{1}{T_I}\int_0^t e\mathrm{d}t + T_D\frac{\mathrm{d}e}{\mathrm{d}t}\right) + u_0$$



Quasi-continuous PID control algorithm（6）

- Digital PID control algorithm

—PID control law with the numerical approximation

—Numerical approximation:

the summation instead of integration,

with the Backward difference instead of differential

analog PID discretized into the differential equation

— Two forms: Positional, incremental

Quasi-continuous PID control algorithm（7）

- The positional PID control algorithm

$$\int_o^t e(t)\mathrm{d}t \approx T\sum_{j=0}^{k} e_j$$

$$\frac{\mathrm{d}e(t)}{\mathrm{d}t} \approx \frac{e_k - e_{k-1}}{T}$$

$$u_k = K_P[e_k + \frac{T}{T_I}\sum_{j=0}^{k} e_j + \frac{T_D}{T}(e_k - e_{k-1})] + u_0$$

Positional control algorithm provides actuator position u_k, cumulative e_k

Quasi-continuous PID control algorithm（8）

- Incremental PID control algorithm

$$u_k = K_P[e_k + \frac{T}{T_I}\sum_{j=0}^{k} e_j + \frac{T_D}{T}(e_k - e_{k-1})] + u_0$$

$$u_{k-1} = K_P[e_{k-1} + \frac{T}{T_I}\sum_{j=0}^{k-1} e_j + \frac{T_D}{T}(e_{k-1} - e_{k-2})] + u_0$$

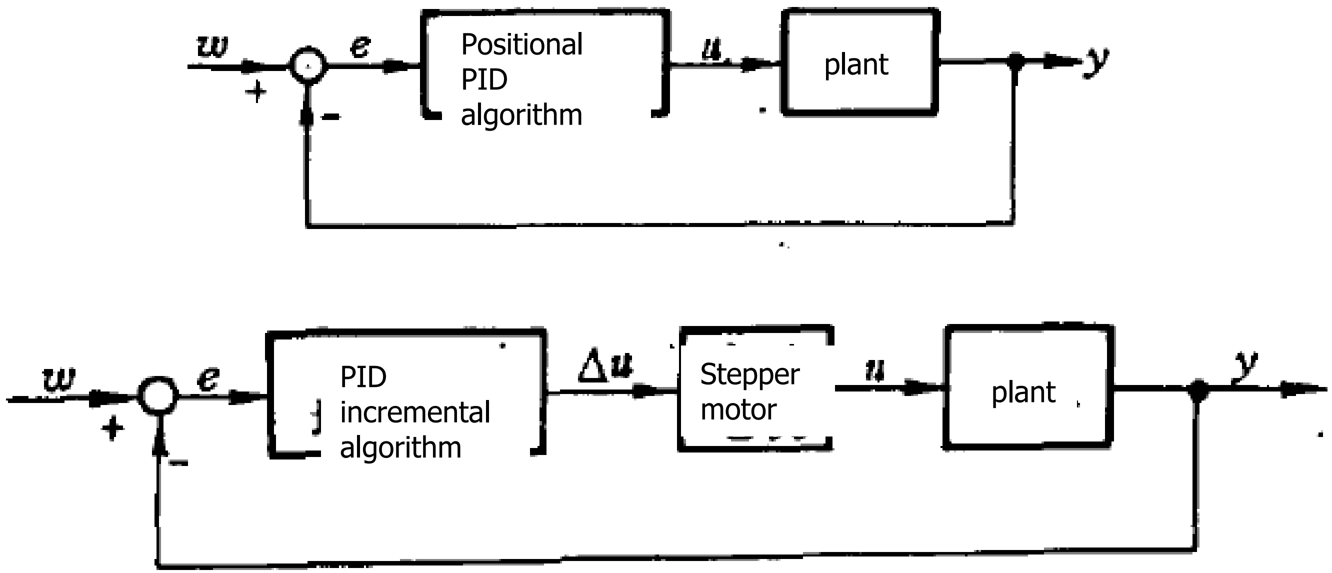
$$\Delta u_k = u_k - u_{k-1} = K_P[e_k - e_{k-1} + \frac{T}{T_I} e_k + \frac{T_D}{T}(e_k - 2e_{k-1} + e_{k-2})]$$

The increment △ uk is feedback to the actuator

just need to keep 3 previous deviation values

Quasi-continuous PID control algorithm（9）

- Positional and incremental PID control algorithm comparison



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Quasi-continuous PID control algorithm（10）

★ Incremental algorithm do not need to accumulate -》 have low inference from the calculation error and accuracy;

positional algorithms use the accumulated value of deviation -> have a bigger cumulative error.

★ control is switched from manual to automatic

the positional algorithm must first set the computer output value as the initial value u0(impact of the switch);

incremental algorithm is independent of the original value (no impact).

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Quasi-continuous PID control algorithm（11）

- The positional PID control algorithm programming
  - Ideas:

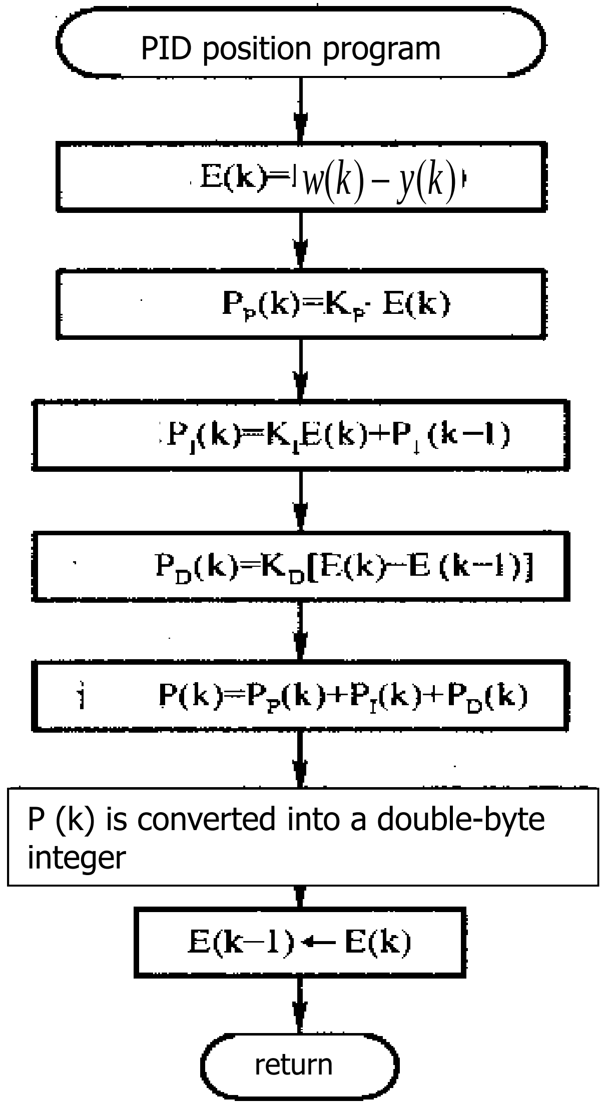
$$u_k = K_P e_k + K_I \sum_{j=0}^{k} e_j + K_D (e_k - e_{k-1})$$

$$P_P(k) = K_P e_k$$

$$P_I(k) = K_I \sum_{j=0}^{k} e_j = K_I e_k + P_I(k-1)$$

$$P_D(k) = K_D (e_k - e_{k-1})$$

$$K_I = K_p T / T_I, \quad K_D = K_p T_D / T$$



Quasi-continuous PID control algorithm（12）

- Incremental PID control algorithm programming

$$\Delta u_k = d_0 e_k + d_1 e_{k-1} + d_2 e_{k-2}$$

Initialization placed adjust the parameters d0, d1, d2, and the set value w, and initial value setting error $e_i = e_{i-1} = e_{i-2} = 0$